Pre-Service Teachers' Pedagogical And Content Knowledge About Trigonometry And Geometry: An Initial Investigation

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In this paper, we report on our initial application of a theoretical model (Nason, Chinnappan & Lawson, 1996) which explicates the relationship between the organization and accessability of teachers' subject-matter knowledge, the nature of their teaching and the nature and quality of student learning within the domain of Analytical Geometry and Trigonometry in a Pilot Study. The major aim of this study was to begin the process of validating the hypothesized relationships and causal connections between the source knowledge elements of the model. This was done by examining the nature of a pre-service student teacher's (a) substantive mathematical knowledge, (b) pedagogical content knowledge, (c) knowledge about the learner, and (d) the relationships and causal connections between these three types of knowledge in the domain of geometry and trigonometry.

Although a significant number of recent research studies have attempted to examine teachers' subject-matter knowledge (see Ball & McDiarmid, 1990 for a review of this research), there has been a dearth of work that has examined the issue of how the teachers' subject-matter knowledge affects the quality of their teaching and the quality of their students' learning.

A corpus of research related to this particular issue has focussed on teacher instructional explanations (e.g., Leinhardt, 1987, 1988, 1989). This research has generated some significant information about the nature of teacher explanations. However, it has not offered a theory of the role of teacher subject-matter knowledge in classrooom instruction nor has it provided a specification of how teacher subject-matter knowledge influences instruction (Leinhardt, Putnam, Stein & Baxter, 1991). It also has provided little information about what kinds of subject-matter knowledge are particularly salient and how subject-matter knowledge is accessed and exploited during the course of teaching and how this in turn affects the quality of students' learning.

In 1995, Nason, Lawson and Chinnappan began the Ramanugan Project. The overall aim of this project is to begin to fill this void by the development of a theoretical model that explicates the relationship between the organization and accessability of teachers' subject-matter knowledge, the nature of their teaching and the nature and quality of student learning within the domain of Analytical Geometry and Trigonometry. In this paper, we present our initial conceptualization of the model and report on an initial investigation in which we investigated a pre-service secondary mathematics teacher's pedagogical and content knowledge about trigonometry and geometry.

Initial Conceptualization of the Model

The initial conceptualization of the model (presented in Figure 1 below) is based on our analysis and synthesis of the research literature (See Nason, Chinnappan & Lawson, 1996). This model (1) identifies key elements that underlie teacher expertise such as substantive mathematical knowledge, pedagogical content knowledge, teachers' perceptions and beliefs about mathematics etc., and (2) attempts to state in graphical form the relationships and causal connections between these elements. A more detailed presentation of this model can be found in Nason, Chinnappan and Lawson (1996).

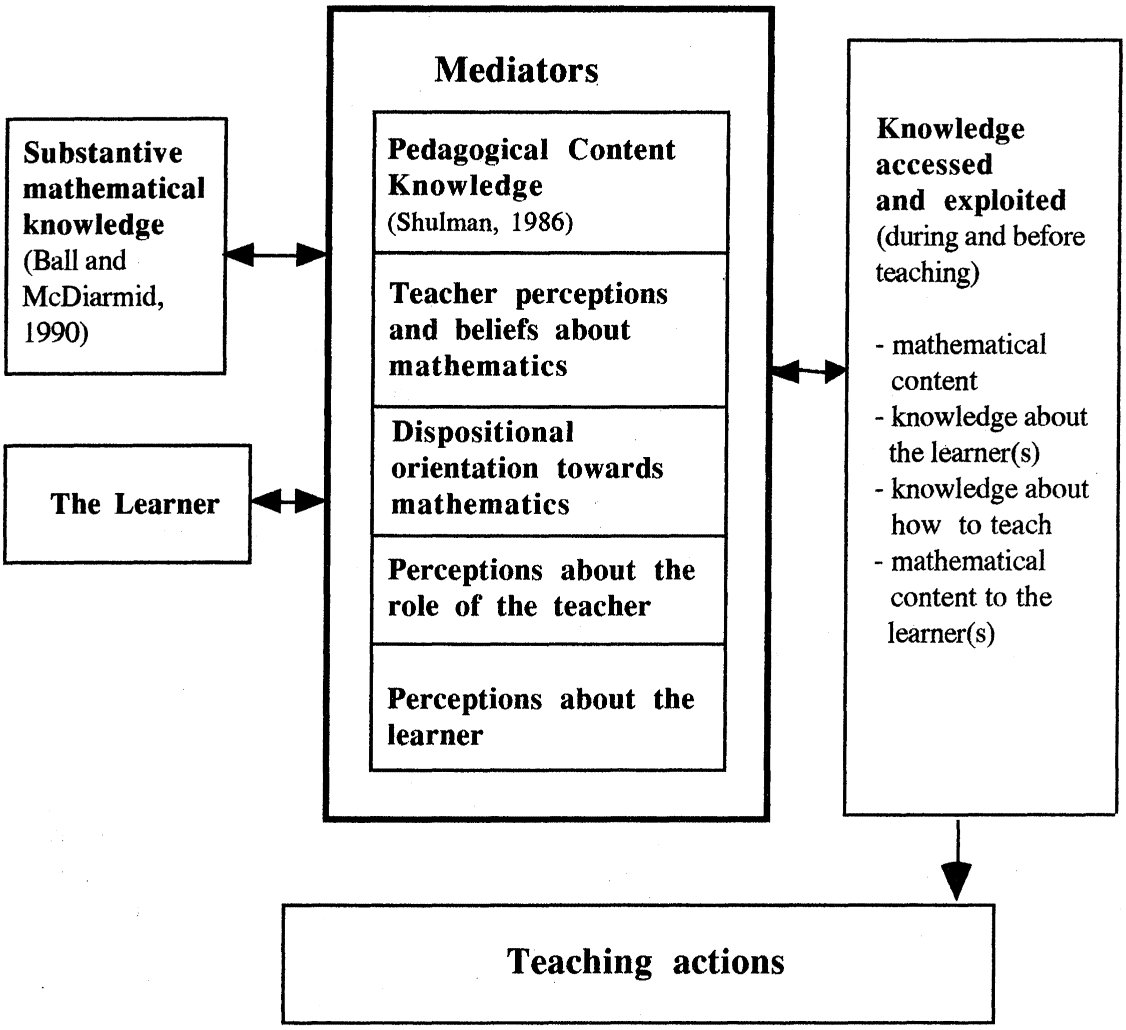

Figure 1

From our initial conceptualization of the model, we have generated the following hypotheses:

(1) that the quality and the quantity of the mathematical content accessed and exploited by teachers both before and during teaching depends to a large extent on the organizational quality of the teachers' repertoires of substantive mathematical knowledge and their reflective awareness of that knowledge;

(2) that the mathematical content accessed and exploited by teachers is related to the teacher's repertoire of pedagogical content knowledge and their dispositional orientation towards mathematics as indicated by the teachers' perceptions and beliefs about mathematics ;

(3) that the quantity and the quality of the knowledge about the learners is related to the teachers' pedagogical content knowledge, perceptions about the learners and perceptions about their roles as a teacher;

(4) that the quality and the quantity of the knowledge about how to teach the mathematical content accessed and exploited by teachers is related to the organizational quality of the teachers' repertoires of pedagogical content knowledge and their reflective awareness of that knowledge ; and

(5) that the knowledge about how to teach the mathematical content accessed and exploited by teachers before and during teaching is related to the teachers' perceptions and beliefs about the role of a teacher and the teachers' perceptions and beliefs about the learners.

In the next section of this paper, we report on our initial examination of the hypothesized relationships and causal connections between the source components shown in our model. This involved an attempt to identify a pre-service student teacher's (a) substantive mathematical knowledge, (b) pedagogical content knowledge, (c) knowledge about the learner, and (d) to map the relationshipss and causal connections between these three types of knowledge in the domain of geometry and trigonometry.

The Study - Method

A third year BEd preservice teacher who was majoring in secondary mathematics and computing(David) volunteered to participate in this study.

One week prior to the interview sessions, the investigator (Chinnappan) and David met for about thirty minutes. At this initial meeting, David was informed that he would be required to talk about concepts related to the areas of geometry, trigonometry and analytical geometry and that he also would be required to respond to question(s) relating the identification of these concepts, the importance of these concepts and how best to teach these concepts to his students. David was invited to raise questions relevant to these issues during this pre-interview meeting and during the course of the rest of the week before two interviews.

The first interview session began with David being asked to talk about all the concepts related to the areas of geometry, trigonometry and analytical geometry that he could recall. As he discussed each of these concepts, the interviewer asked follow-up questions in order to probe the width and depth of David's knowledge about each of the concepts and also to identify the degree of connectedness between these concepts. Then David was presented with an analytical geometry problem and asked to solve the problem out aloud. This problem task was administered in order to ascertain how David accessed and interconnected his repertoire of geometrical concepts in a problem-solving context. After he had completed the problem, he was asked to investigate other ways of solving the problem. The probing for alternative solutions focussed on the flexibility, the depth and the interconnectedness of his repertoire of concepts.

During the second interview, David was first reminded about mathematical concepts that had been identified and discussed during the first interview. Then he was asked to describe how he would teach those concepts. As this interview progressed, the investigator (Nason) frequently asked probing questions in order to get David to elaborate on and/or clarify points he had made about the mathematical content and his perceptions about learners and about how he would teach the mathematical concepts.

Both interviews were video- and audio-recorded and transcribed. The analysis of the data from Interview 1 focussed on the major concepts that David identified in relation to geometry and trigonometry, and the relations or links that he was able to construct. The analysis of data from Interview 2 focussed on David's pedagogical content knowledge and his knowledge and perceptions about learners. The initial analysis of the data was done by two of the investigators. The conclusions drawn from their analysis of the data were confirmed by having the third investigator at a later date go through the video-recordings and transcripts of the two interviews looking for negative evidence. As Sowden and Keeves (1990) point out, while the failure to find negative evidence after a deliberate search does not and cannot establish the "truth" of a conclusion, it however, does increase the probability that the original conclusion is sound. Some modifications were made to the set of conclusions following this process of verification.

Results

Substantive Mathematical knowledge

In this section, we present the results of our analysis of David's response to the investigator's questions about the identification of concepts in geometry, trigonometry and analytical geometry, and other concepts that could be related to these three areas.

Table 1 shows a list of the major concepts activated by David. In broad terms, on these topics of geometry, trigonometry and analytical geometry, one can isolate four major concepts: gradient, trigonometric ratios, similar triangles and Pythagoras theorem. It is interesting to note that David considered the notion of gradient as a starting point. His explanation of this concept included a procedure for working out the gradient of two points under two situations: when the coordinates of the points were not given, when the coordinates of the points were given. David also explained how a numerical value of gradient could be interpreted. For instance, he attempted to illustrate the meaning of a line with a gradient of 1/4 using the notions of vertical and horizontal distances. This idea was subsequently related to one of trigonometric ratios, namely the tangent ratio.

In this context , David quite appropriately mentioned how the idea of tangent of an angle could be used to identify similar triangles. He did mention the sine and cosine ratios but did not explain how the gradient and tangent ratio could be utilised in the derivation of sine and cosine ratios. That is, the link between tangent of the angle, gradient of the line and sine/cosine of the angle was not made explicit.

David then moved on to Pythagoras' theorem for which he gave the algebraic statement (a2+b2=c2). Although he did provide an alternative way to understand the

CONCEPTS IDENTIFIED	LINKS IDENTIFIED	LINKS NOT MENTIONED
Geometrical shapes		Relationship between two dimensional figures and their properties
Mensuration		Relationships between the concepts of area and perimenter; Areas and perimeters of geometrical figrues
Cartesian coordinate system	coordinates of a point, vertical and horizontal distance; gradient and tangent ration of angle	Coordinate system and analytical geometry; Coordiante system and graphing of polynomials
Gradients	Gradient, tangent ratioand similar triangles	Gradient and sine or cosine ratio
Trigonometric ratios	Tangent and gradient	Relationship between tangent, sine and cosine ratios
Pythagoras theorem	Pythagoras' theorem and areas of squares	Pythagoras' theorem and trigonometric identities; Pythagoras' theorem and coordinate system; Pythagoras' theorem and the right-angled triangle
Linear functions	functions and calculators; link between the x and y variables	Linear functions and gradient; linear functions and coordinates of points; graphing of linear functions
Equations	subtitution of numerical values into an equation	Equations and functions; equations and Pythagoras' theorem; solution of system of equations

Table 1

algebraic statement by way of considering areas of squares that could be constructed by using the three sides of the right-angled triangle, there is no evidence of David making any links between this representation and other related concepts such as the Pythagorean triplets, the converse of Pythagoras theorem. David could have also used this relationship in the context of a coordinate system to work out for instance the radius of a circle with its centre located at the origin, but he did not do so showing a lack of understanding of application of this idea in a range of situations.

The list in Table 1 shows that David had built up a range of mathematical concepts in relation to the topic of geometry, trigonometry and analytical geometry. However, he did not make or attempt to establish any meaningful mathematical connections between the various areas despite instructions to do so. This pattern suggests that his mathematical content knowledge base lacks a high degree of organisation and integration.

Pedagogical content knowledge

Interview transcripts were analysed for instances of activation of pedagogical content knowledge (Shulman, 1986). This analysis led to the identification three subgroups of knowledge within this category:

1) Pedagogical content knowledge showing the need for historical background;
(2) Pedagogical content knowledge showing the need for discussing application; and
(3) Pedagogical content knowledge showing the need for using particular method.

The need for historical background

When asked to explain how he would go about teaching some of the concepts mentioned in the first interview, David showed an interest in getting his students to become aware of the history of certain concepts. For instance, in relation to the teaching of the Pythagoras theorem, David made the following remark:

> *I would probably the night before the lesson do a little bit of research on Pythagoras and probably establish a bit of history so they know a little bit of (about) where it originated and why this formula (c2 = a2+b2) is used right throughout mathematics.*

The importance of application

David was also interested in sharing with this students the applications of mathematics. He mentioned that Pythagoras' theorem is used to solve different geometrical problems and that engineers also use it in their work. In response to a question about the importance of studying gradient, David's reply was:

> *Well I've had experience in surveying and to me that's totally important if you get a job with a surveyor*

Pedagogical content knowledge showing the need for using particular method

In the examination of teachers' pedagogical content knowledge, we were interested to find out about David's understanding of various methods he would employ in presenting ideas and initiating discussions about these ideas. In this context, David's remark that he would solve the problems on the chalkboard (*'I can do problems for them on the board'*) is interesting. How to intrepret this comment is not yet clear - it could reflect merely his observation that he had observed his own teachers or lecturers doing this, or it could represent a view that it is important for the teacher to model the problem solving process for students- this indicates an area where further probes are necessasy in the interview

David favoured the use of examples to teach mathematical concepts. For example, in one part of the interview, David was asked about teaching the concept of linear function. He selected one example of this class of function and decided to explain it. At the end of this explanation, David remarked that :

Alright let's put in in the form of that, so it's going to be, may be I'll pick another example, it might be easier. Something like, alright let's say something in the form of this

The above reaction indicated that David did consider the use of examples to an effective and appropriate instructional strategy for teaching linear functions. This part of his pedagogical content knowledge also shows that care needs to be exercised in the selection of examples so that the more simpler one are used first.

While the above points highlight that David did attend to components of pedagogical content knowledge as theorised by our model, such instances were too few to have any significant bearing on his teaching. Analysis of the rest of the interview showed that David's knowledge of geometry and trigonometry, and how to teach these concepts was diffuse. At the end of his interviews, we were unable to clearly determine how his available mathematical knowledge would be deployed during teaching.

Knowledge about the Learner

As shown in Figure 1, a teachers' understanding of and assumptions about the learner exerts a major influence in the learning process. Towards this end, we were interested to examine whether and to what extent David showed consideration for the characteristics of his students. Our analysis showed that there was very little talk about the learner, or about the requirements of the learner, or of what learners could be doing during lessons while all the content is being presented.

What discussions there was of the learner referred only indirectly to knowledge of the learner, but not at all to the actions that a learner might engage in to develop an understanding of this content. For example, David mentioned that in the teaching of geometry, he will start with two dimensional shapes before investigating other concepts such as finding the areas and perimeters of such shapes. This seems to be a reasonable approach, but there is no evidence of how he would help the learner move from understanding properties of two-dimensional figures to working out their areas and perimeters.

During the interview, David made several assumptions about the learner. For example, when asked to explain how he would teach the notion of functions, David observed that it would better to start with an example that might be easier, perhaps indicating some appreciation of cognitive load.

'maybe I'll pick another example,it might be easier'.

He was also aware that some students might find certain concepts harder to process but does not explain why this is so. Several times, he made the comment that

'if they understand they will feel comfortable'.

This observation implies that helping students understand the concepts would engender a feeling of satisfaction and confidence in dealing with similar concepts.

What was not mentioned about the learner

In analysing the interview transcripts, we found that David did not talk about a number of characteristics about the learner. Firstly, there was no indication about the desire to understand the beliefs his students might hold about geometry or trigonometry. Secondly, students' dispositions towards this area of mathematics did not get any mention. David also did not consider a) the actions that the learners can engage in, b) the strategies that can be used by the learner, and c) the different ways of problem solving. In general, David's approach suggested that beyond the presentation core concepts, the teacher plays very little further role in the learning of geometry and trigonometry.

Discussions and Conclusions

A principal concern of the Ramanugan Project is to examine the nature of teachers' mathematical content knowledge, how this knowledge is structured for the purposes of teaching mathematics concepts, principles and procedures, and the assumptions teachers make about their students. A preliminary evaluation showed that the framework set out in Figure 1 is useful for classifying the contents of a teacher's talk.

The model has provided us with a useful tool for analysing some of the ways in which characteristics of a student teacher's knowledge might be different from those argued to be characteristic of expert teachers. One area of this difference involves the extent and coherence of mathematical content knowledge. As expected, this student teacher showed acceptable levels of this knowledge. However, the components of this knowledge were found to be relatively discrete. There was little evidence that the knowledge components were connected in a manner that would lead to the establishment of more extended networks of mathematical knowledge that are argued to be characteristic of expert teachers mathematical knowledge base (Shulman, 1986).

The type and sequence of mathematical concepts activated by our student teacher suggests that he had not reflected upon these in order to construct multiple representations for these concepts, an important requirement for the effective activation and of use the concepts in the solution of novel problems (Larkin, 1979; Shoenfeld, 1985).

The data analysed here indicated that, despite our instructions to highlight the various relationships among knowledge components, the student teacher merely retrieved isolated bits of mathematical information without any consideration to how they could anchored within the general mathematical curriculum, a point that is stressed in current documents about effective mathematics teaching (National Council of Teachers of Mathematics, 1989). This behaviour suggests that there is the possibility of the students of this student teacher being 'bombarded' with unrelated, or poorly related, mathematical information.

The knowledge base of the student teacher also showed a general lack of integration of mathematical knowledge with pedagogical principles. There was little evidence that the student teacher had considered the possibility of reconceptualising or restructuring the mathematical content knowledge in preparation for presentation to students. There was also no sense that this novice teacher was concerned to engage his students in active exploration of the subject-matter knowledge. The teacher's pedagogical content knowledge seemed to be poorly developed.

The third key feature of our student teachers' knowledge was that the role of the learner was under-conceptualised. Although there are instances where he had shown consideration to the learner in terms of how he or she would handle the concepts, there is a lack of understanding about how students activate and exploit mathematical knowledge acquired in the classroom.

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